## IMF Working Paper

# Lending Resumption After Default: Lessons from Capital Markets During the $19^{\text {th }}$ Century 

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# IMF Working Paper 

Monetary and Financial Systems Department

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#### Abstract

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This paper mines the experience of capital markets during the 19th century to propose an alternative way of interpreting international default episodes. The standard view is that defaulting on sovereign debt entails exclusion from capital markets. Yet we have observed multiple instances of sovereign debt default in which the reaction of lenders was not the one predicted by the punishment story: in some cases, lending ceased for long periods, but in others it was not interrupted. This paper claims that the reaction of lenders after default stems from the additional knowledge about the borrower that lenders acquire during these episodes. The lending relationship is modeled in a costly state-verification environment in which governments have private information about their investment projects (good or bad). It is shown that, in the event of default, it is worthwhile for lenders to find out more about the type of project, and then interrupt lending only if the project is believed to be a bad one.


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## I. Motivation

One of the main differences between international loans and domestic loans is that international repayments are harder to enforce. As a result, the international finance literature has devoted considerable attention to the question of what determines a sovereign nation's willingness to repay its loans. The common wisdom in the literature is that borrowers will repay if faced with severe punishments for not doing so. Thus, creditors should exclude a defaulting nation from international capital markets forever. ${ }^{2}$ This harsh punishment is designed to induce nations always to repay, and thus it is in the interest of lenders to treat all defaulters alike. Nevertheless, a close look at historical episodes of default reveals that punishment in the form of exclusion did not always follow default. In practice, the reaction of lenders to default varied widely across countries, with some denying fresh credit for long periods, but others granting new funds soon after default. As discussed in the next section, the lender's discrimination in granting new loans was based on the perceived future profitability of the borrower's activities.

A complete theory of international lending should comprise these two key elements: (i) the inability to enforce contracts, which requires the existence of punishments for breaking agreements; and (ii) the differential treatment received by defaulting nations as an optimal response of lenders. The literature has been quite successful in rationalizing why exclusion from future lending is needed when contracts are not enforceable. However, less effort has been devoted to developing models in which lenders discriminate among types of defaulters. This paper tries to make a contribution to the second aspect. That is, assuming enforceability of contracts, I study an environment in which it is optimal for lenders to treat defaulting nations differently based on knowledge acquired during default. In particular, I consider an environment in which there are different types of borrowers (governments). Each type of borrower is identified with a specific investment project (good and bad), whose return is private information. However, lenders are able to observe the realized return of the project, as well as a signal about the type of project, if they pay a small auditing cost. I show how, in this environment, after default, the reaction of lenders is not necessarily to deny fresh credit, and that, indeed, it is easier for a government with good projects to obtain new credit than it is for a government with bad projects.

The paper most similar to this one in the international finance literature is Cole, Dow and English (1995). These authors explain how countries that have been denied credit at a given point in time will eventually be readmitted to credit markets. This paper, on the other hand, explains why some nations are not denied fresh credit even when they have just defaulted on their debt. Grossman and Van Huyck (1988) present a model in which a government borrows to finance investments. The government signs a state-contingent contract with the lender. This contract specifies that in high states of nature the government should repay an amount

[^1]higher than the value of the borrowed funds times one plus the interest rate, whereas in bad states, the government payments are less than this amount. This last event is what Grossman and Van Huyck call partial default-which does not lead to any punishment. Only full repudiation of payments entails permanent exclusion from capital markets. But this is not based on any learning by lenders. Additionally, all repudiations are treated alike. Another important difference between their paper and this one is that I derive the form of the optimal contract, whereas they take it as given.

Another prominent work in this literature is Atkeson (1991). In Atkeson's model, as well as in the present paper, uncertainty about projects and the risk of default constitute impediments to writing international lending contracts. However, the results I present differ from those in Atkeson (1991) in some important aspects. In particular, Atkeson focuses on allocations that are free of risk of repudiation and tries to explain why it has been observed that some countries experience a capital outflow when the worst realizations of national output occur. In this paper, on the other hand, a rationale is offered for the observed pattern of default, subsequent verification of investment levels, and possible lending resumption.

This paper also differs from Diamond (1989) in several respects. First, in Diamond's model, good projects (i.e., "safe" projects in Diamond's terminology) never default, and so, if a borrower ever defaults, then that borrower is excluded from the credit market forever since it becomes known that the project is a bad one (i.e., "excessively risky"). As argued above, we have not observed this pattern empirically. Additionally, another prediction of Diamond's model is that as time goes by, those borrowers that have access to good and bad projects will start undertaking only the good ones. This is so because as time evolves, the benefits of having a good reputation increase: a good track record of no defaults results in lower interest rates. Thus, we should observe over time that borrowers (nations) tend to choose the safer projects and never default. A simple look at the long history and pervasiveness of default episodes in some countries suffices to suggest that, indeed, borrowers do not converge to this "choosing the good project" behavior predicted in Diamond (1989).

This paper is structured as follows. The next section presents and discusses some historical evidence on default episodes. Section III sets the stage by presenting a model of lending with costly state verification and private information about a government's type of investment project and its realized return. As a preliminary step for obtaining the main results, Section IV examines the case in which the type of government is publicly known. In this section it is proved that the form of the incentive-compatible contract is a standard debt contract in which default and verification only occur when the reported state of the economy is low. Section V derives a similar result in the more general case in which types are unobservable. Section VI discusses the applicability of the model to the historical experience under consideration. The Appendix contains proofs for some results of the paper.

## II. Historical Evidence on Default and Lending Resumption

This section reviews some of the main events observed in international bond markets during the period 1820-1920. My recounting of these events draws heavily from Borchard (1951),

Fishlow (1985), and Wynne (1951). The period 1820-1920 was chosen because the international capital market was already well developed and because it provides a scenario isolated from third-party interference (such as that of the public international financial institutions). This last element is a crucial one which takes into account the fact that the nature of the contracts written between lenders and borrowers is unavoidably affected by the institutional framework in which the contract is embedded. In this sense, third-party institutions that do not explicitly intervene in the drafting of a contract may indirectly affect the incentives of the parties involved in the contract. Think for instance of the role attributed to the International Monetary Fund and other organizations in the international capital markets. Some have argued that the existence of the IMF and its rescue packages have distorted the incentives of borrowing nations to invest in good projects, or even those of lenders, making them less zealous in their financing decisions. Since the goal of this paper is to study international lending in an environment in which these forces are not present, I will not consider the post-Second World War period. Additionally, given that the theme of the paper is lending resumption, the period spanning from 1930 to the early aftermath of the Second World War is not covered either, for in those years creditors were indiscriminate in their denial of fresh credit. As explained by Lindert and Morton (1989), developing nations often had difficulties in obtaining funds, regardless of whether they had previously defaulted or not. These were years of dry capital markets.

Despite the risk of repudiation and the lack of means to monitor investments, prior to 1914 most of the international lending took place via bonds rather than bank loans. Lending was primarily channeled from private creditors based in Europe to sovereign governments around the globe. Whenever defaults occurred, they were typically associated with bad states of the economy, such as a substantial drop in the export revenue of the defaulting country. Examples of this abound. For instance, Brazil defaulted in 1898 after a decline of 64 percent in coffee prices over the preceding five years. In 1914 it defaulted again after a fall of almost 40 percent in coffee prices in two years. Similarly, other Latin American countries descended into bankruptcy as a result of the fall in the prices of guano and other raw materials. Similarly, Bulgaria was driven to the brink of bankruptcy in 1899 by a series of bad harvests.

The reaction of lenders to default varied. For some of the defaulting nations we observe that lending resumed shortly after default, sometimes even under concessionary terms and before repayments started. This was the case, for instance, for Argentina in 1891, Brazil in 1898 and 1914, Mexico in 1914, and Bulgaria in 1900. In other cases, defaulting nations were not able to obtain new funding for long periods of time. This was so for most Latin American countries in the 1820s, but also for Turkey in 1875, Egypt in 1876, Peru in 1876, and Greece in 1893. Perhaps, the two most strikingly opposite examples are those of Argentina and Peru. Argentina declared bankruptcy in November 1890 due to a decrease in export earnings. However, lending resumed in March 1891. On the other hand, Peru defaulted on its sovereign debt in 1876, and did not obtain fresh credit until 1889. The distinction made by lenders was primarily based on the type of investments that the governments of these countries had been undertaking: when lenders deemed that countries had been investing in infrastructure and other projects that enhanced their future growth, new loans were relatively
easy to obtain, whereas for countries that used the borrowed funds to sustain current consumption and repay older debts, further credit was denied.

Another case in point is Bulgaria in the 1890s. Bulgaria had contracted large loans in 1892 and 1896. These funds were raised to undertake the construction of railways and harbors, as well as to develop a series of agricultural banks. After some years of budgetary deficits, the situation became aggravated by a succession of bad harvests and the country was put on the verge of default in 1899. However, at the beginning of 1900, foreign funds were sought and a syndicated loan, headed by the Banque de Paris et des Pays-Bas, was agreed upon. ${ }^{3}$

Typically, after default lenders conducted some type of review of the past investment behavior of the borrowers, and lenders were granted rights over some of the tax revenues of the defaulting government (see Borchard, 1951, for a detailed discussion). Interestingly enough, default also entailed some form of financial control that the lenders imposed on borrowers. This was in order to secure the profitability of the pledged tax revenues, as well as, in the case of the nations granted new credit, that of the renewed investments (Borchard, 1951, and Fishlow, 1985). Financial control was imposed in a wide variety of ways. It ranged from control of the monetary and fiscal authorities of the country (e.g., quantity of money in circulation ${ }^{4}$, and types of taxes to be levied), to the simple inspection of the books and accounts kept by the agencies in charge of the pledged revenues as compensation for the defaulted loans.

To summarize, the salient features the paper focuses on in the next section are that (i) default took place during bad states of the economy, (ii) default episodes were periods in which lenders revised their knowledge of borrowers, and (iii) lenders dispensed different treatment to defaulting borrowers, depending on whether they were perceived as profitable in the future.

## III. The Environment

Consider a two-period economy in which there is a government that is alive for two periods. The government has access to an investment project that can be either of two types (good or bad). The government cannot choose the type of its investment project: it is given by nature. Thus a government is identified with the project it has access to. Throughout the paper, I will dub as "a good government," a government with a good project, and "a bad government," a government with a bad project. The specific type of project is private information of the government. In order to operate its investment project, the government needs to borrow funds

[^2]from a lender. There are two one-period lenders, each one being alive for a different period. In other words, there is a lender alive in period one and a lender alive in period two.

Each period is subdivided into three subperiods (see Figure 1). At the beginning of each period the international lender is born endowed with $x$ units of consumption good as well as (publicly known) prior about the type of government. The government designs a contract and offers participation in it to the lender. The lender can either loan its endowment to the government or store it until the moment of consumption. Immediately after this, investment takes place. During the second subperiod, the random return of the project, $y_{t}$, is realized and the government makes an announcement $m_{t}$ (message) about the realized value, $y_{t}$. The realized return of the project is private information of the government. Nevertheless, the lender can observe it by paying a small verification cost, $\mu$. During verification, the lender also receives a noisy signal, $s_{t}$, about the type of government. Verification takes place during the third subperiod. Also during this last subperiod, the government makes repayments and agents consume. Finally, the lender updates his beliefs based on all the information made public during the period. The updated beliefs are publicly known, and thus are available to the second-period lender. Then the second and last period begins.


Figure 1: Timing of the Model

Potentially, the elements of a contract depend on all the history available to both agents, government and lender, at the time the contract is written. This history may include the messages sent by the government, $m_{t}$, the signals received, $s_{t}$, and the realized values of $y_{t}$. All this information can be condensed (using Bayes' rule) to deliver a probability, or prior, that the government is good.

## A. The Government

As mentioned, the government can be of two different types, good and bad, denoted by $\theta \in\{G, B\}$, respectively. The good type of government has access only to an investment
project that requires an investment of $x$ units of the consumption good per period. Returns of this project are

$$
y_{t}=\left\{\begin{array}{l}
y^{h} \text { with prob. } \pi  \tag{1}\\
y^{l} \text { with prob. } 1-\pi
\end{array}\right.
$$

where $y^{h}>y^{l}$. The bad government, on the other hand, has access only to an investment project that requires a smaller investment, $\gamma x(\gamma<1)$, but that delivers $y_{t}=y^{l}$ with probability one. Both types of project deliver $y_{t}=0$ if there was no investment. Let $i_{t}{ }^{\theta}$ denote the investment level of government $\theta$ at time $t$. Invested amounts are only observable by the government.

Realizations, $y_{t}$, of the projects are private information of the governments, and it is assumed that

$$
\begin{gather*}
\pi y^{h}+(1-\pi) y^{l}-\mu>x  \tag{2}\\
p x>y^{l} \tag{3}
\end{gather*}
$$

where $\mu$ is a small positive number. The role of these two assumptions is to introduce a tension between borrowers and lenders. Assumption (2) states that, in expected value, it is worthwhile to invest in the good project even if verification takes place in all states. On the other hand, assumption (3) states that the bad project is not worth undertaking at all.

Instant utilities for the governments are given by

$$
\begin{equation*}
u_{t}^{\theta}=E\left(c_{t}^{\theta}\right), \mathrm{t}=1,2 \tag{4}
\end{equation*}
$$

Where $c_{t}{ }^{\theta}$ is the amount available to the government for consumption at time $t$. The budget constraint of the government is

$$
\begin{gather*}
i_{t}^{\theta}=b_{t}  \tag{5}\\
c_{t}^{\theta}+i_{t}^{\theta}=y_{t}-p_{t}+b_{t} \tag{6}
\end{gather*}
$$

where $b_{t}$ is the amount borrowed from international lenders and $p_{t}$ stands for any repayments owed to lenders and will be specified in detail below. I assume that once funds are lent, there is a technology that forces governments to invest the minimum necessary to run the projects. ${ }^{5}$ In other words $i_{t}{ }^{G}=x$ if and only if $b_{t} \geq x$; and $i_{t}{ }^{B}=\gamma x$ if and only if $b_{t} \geq \gamma x$. For simplicity, let both types of government discount the future at the same rate $\beta=1$.

[^3]
## B. Lenders

In every period, a one-period-lived international lender is born. This lender is endowed with $x$ units of the consumption good and with some information, or beliefs, about the type of government in office. The lender can store his endowment until the end of his life, or loan it in return for part of the investment return, $p_{t}$. Just before dying, the lender is allowed to consume.

The lender is able to observe the true realization of $y_{t}$ only if he pays a verification cost of $\mu$. In that case, he also receives a signal, $s_{t} \in\{G, B\}$, about the type of government. The signal is informative but not perfect, and it does not depend on the realization of $y_{t}$. More specifically,

$$
\begin{equation*}
\operatorname{Pr}(s=G \mid \theta=G)=\operatorname{Pr}(s=B \mid \theta=B)=q>1 / 2 . \tag{7}
\end{equation*}
$$

That is, the probability of getting the correct signal about the type of government is greater than one-half. Assume also that

$$
\begin{equation*}
(1-\pi) q>1-q \tag{8}
\end{equation*}
$$

In other words, the informative power of the signal $s$ is high enough relative to the informative power of the observation $m_{t}=y^{l}$. That is, if a low realization is observed along with a good signal, the probability of this event arising from a good government (i.e., $(1-\pi) q)$ is higher than the probability of it arising from a bad one (i.e., $1-q)$.

Let $d_{t}$ be an indicator function for the verification decision of the lender. Let the value $d_{t}=1$ indicate that verification takes place.

The utility function of the lender is

$$
\begin{equation*}
u^{L}=E\left(c_{t}^{L}\right) \tag{9}
\end{equation*}
$$

where $c_{t}{ }^{L}$ denotes the lender's total consumption at the end of his lifetime and must satisfy the lender's budget constraint

$$
\begin{equation*}
c_{t}^{L}=\left(x-b_{t}\right)+p_{t}-\mu d_{t} \tag{10}
\end{equation*}
$$

Definition 1: A public history $h^{t}=\left(h_{\tau}\right)_{\tau=1}^{t}, t=1,2$, is a vector for which $h_{\tau}=\left(g_{\tau}, d_{\tau} s_{\tau}\right)$, $g_{\tau}=\left(1-d_{\tau}\right) m_{\tau}+d_{\tau} y_{\tau}$, and where $y_{\tau}, m_{\tau} \in\left\{y^{l}, y^{h}\right\}, d_{\tau} \in\{0,1\}$, and $s_{\tau} \in\{B, G\}$, for $\tau=1,2$.

Note that $s_{2}$ is not relevant, because the model is a two-period model. Given history $h^{t}$, lenders derive a belief about the type of government in office. This belief is constructed through Bayes' rule. Let $\rho_{t-1}$ denote the probability assigned at the beginning of time $t$ to the government in office being of the good type (the initial belief $\rho_{0}$ is given). Based on this prior, $\rho_{t-1}$, the contract signed at the beginning of period $t$ specifies:

- amount lent $b_{t}\left(\rho_{t-1}\right)$; and
- a repayment function $p_{t}\left(\rho_{t-1}, y_{t}, m_{t}\right)$, conditional on previous history and on realizations and messages sent by the government; and
- an auditing rule, identified with an indicator function $d_{t}\left(\rho_{t-1}, m_{t}\right) \in\{0,1\}$, conditional on messages sent by the government and previous history; zero indicates no verification.

The elements of a feasible contract must also satisfy the following restrictions:

$$
\begin{equation*}
p_{t}\left(\rho_{t-1}, y_{t}, m_{t}\right) \leq y_{t} \tag{11}
\end{equation*}
$$

(i.e., the set of truth-telling payments specified in the contract must be feasible), and

$$
\begin{equation*}
d_{t}\left(\rho_{t-1}, m_{t}\right)=0 \Rightarrow p_{t}\left(\rho_{t-1}, y^{l}, m_{t}\right)=p_{t}\left(\rho_{t-1}, y^{h}, m_{t}\right) \text { for } m_{t} \in\left\{y^{l}, y^{h}\right\} \tag{12}
\end{equation*}
$$

(i.e., payments cannot be conditioned on unobserved objects).

Definition 2: Given $b_{t}\left(\rho_{t-1}\right) \in[0, x]$ and $\theta \in\{B, G\}$, a consumption allocation $\left(c^{L}{ }_{t}, c^{\theta}{ }_{t}\right)$, $t=1,2$ is feasible if and only if

$$
\begin{gather*}
c^{L}{ }_{t}+c^{\theta}{ }_{t}+i^{\theta}{ }_{t} \leq x+y_{t}-d_{t}\left(\rho_{t-1}, m_{t}\right) \mu  \tag{13}\\
i^{\theta}{ }_{t} \leq b_{t}\left(\rho_{t-1}\right)  \tag{14}\\
c^{L}{ }_{t}, c^{\theta}{ }_{t} \geq 0 \tag{15}
\end{gather*}
$$

for realized $y_{t} \in\left\{y^{l}, y^{h}\right\}$ and messages sent $m_{t} \in\left\{y^{l}, y^{h}\right\}$ for $t=1,2$.

## IV. Observable Types

If the type of government is observable, beliefs about the type of government in office are unnecessary (and hence they can be dropped in order to lessen notation). In this case, it is clear that the lender does not loan to the bad government since, by (3), the net expected present value of the investment is negative. On the other hand, the lender may be willing to write a contract with the good government.

A feasible incentive-compatible contract maximizes the utility of the government subject to incentive compatibility and participation constraints (described below), as well as feasibility of payments (11), informational constraints (12), and feasibility conditions (13)-(15). In the following proposition, I show that in each period the lender and the government will sign a standard debt contract. ${ }^{6}$ In this contract the government offers to pay a fixed amount $p_{t}\left(\rho_{t-1}, y_{t}, y^{h}\right) \leq y^{h}$ if the announced state of the economy is high $\left(m_{t}=y^{h}\right)$, and will

[^4]default if the announced state is low $\left(m_{t}=y^{l}\right)$. In case of default, the lender will conduct an audit and may seize the realized value of the project: $p_{t}\left(\rho_{t-1}, y_{t}, y^{l}\right) \leq y^{l}$.

Proposition 1: For each $t \in\{1,2\}$, the feasible incentive-compatible contract between the good government and a one-period lender is a standard debt contract. On the other hand, the incentive-compatible contract between the bad government and a one-period lender specifies $b_{t}=0$.

Proof: Consider first the problem of deriving the incentive-compatible contract for the second period. That is, one wants to maximize the expected utility of the government

$$
\begin{equation*}
\pi c_{2}{ }^{G}\left(y^{h}, y^{h}\right)+(1-\pi){C_{2}}^{G}\left(y^{l}, y^{l}\right) \tag{16}
\end{equation*}
$$

subject to informational constraints,

$$
\begin{align*}
& d_{2}\left(y^{h}\right)=0 \Rightarrow p_{2}\left(y^{h}, y^{h}\right)=p_{2}\left(y^{l}, y^{h}\right)  \tag{17}\\
& d_{2}\left(y^{l}\right)=0 \Rightarrow p_{2}\left(y^{l}, y^{l}\right)=p_{2}\left(y^{h}, y^{l}\right) \tag{18}
\end{align*}
$$

incentive compatibility of the government,

$$
\begin{align*}
& y^{h}-p_{2}\left(y^{h}, y^{h}\right) \geq y^{h}-p_{2}\left(y^{h}, y^{l}\right)  \tag{19}\\
& y^{l}-p_{2}\left(y^{l}, y^{l}\right) \geq y^{l}-p_{2}\left(y^{l}, y^{h}\right) \tag{20}
\end{align*}
$$

participation constraint of the lender,

$$
\begin{equation*}
\pi c_{2}^{l}\left(y^{h}, y^{h}\right)+(1-\pi) c_{2}^{l}\left(y^{l}, y^{l}\right)=x \tag{21}
\end{equation*}
$$

and feasibility,

$$
\begin{gather*}
c_{2}{ }^{G}\left(y_{2}, y_{2}\right)+c_{2}{ }^{l}\left(y_{2}, y_{2}\right)+i^{G}{ }_{2}=x+y_{2}-d_{2}\left(y_{2}\right) \mu  \tag{22}\\
i^{G} \leq b_{2} \tag{23}
\end{gather*}
$$

Since we are interested in the case for which there is lending to the good government, set $b_{2}=x$, and thus,

$$
\begin{equation*}
c_{2}{ }^{G}\left(y_{2}, y_{2}\right)=y_{2}-d_{2}\left(y_{2}\right) \mu-c_{2}^{l}\left(y_{2}, y_{2}\right) \tag{24}
\end{equation*}
$$

which can be substituted in the objective function above,

$$
\begin{equation*}
\pi\left[y^{h}-d_{2}\left(y^{h}\right) \mu-c_{2}{ }^{l}\left(y_{2}, y_{2}\right)\right]+(1-\pi)\left[y^{l}-d_{2}\left(y^{l}\right) \mu-c^{l}{ }_{2}\left(y^{l}, y^{l}\right)\right] \tag{25}
\end{equation*}
$$

Rearranging delivers,

$$
\begin{equation*}
\pi y^{h}+(1-\pi) y^{l}-\pi c_{2}\left(y^{h}, y^{h}\right)-(1-\pi) c_{2}^{l}\left(y^{l}, y^{l}\right)-\left\{\pi d_{2}\left(y^{h}\right) \mu+(1-\pi) d_{2}\left(y^{l}\right) \mu\right\} \tag{26}
\end{equation*}
$$

But note that $y^{h}, y^{l}$, and $\pi c_{2}{ }^{l}\left(y^{h}, y^{h}\right)+(1-\pi) c^{l}{ }_{2}\left(y^{l}, y^{l}\right)=x$ are constants, and hence can be dropped from the maximization problem, which now has as an objective function

$$
\begin{equation*}
\pi d_{2}\left(\rho_{-1}, y^{h}\right) \mu+(1-\pi) d_{2}\left(\rho_{-1}, y^{l}\right) \mu \tag{27}
\end{equation*}
$$

subject to the same constraints. The incentive-compatible contract at the second stage is the one that minimizes verification costs.

Suppose now that $d_{2}\left(y^{l}\right)=d_{2}\left(y^{h}\right)=0$. Then, since $p_{2}\left(y^{h}, y^{l}\right)=p_{2}\left(y^{l}, y^{l}\right) \leq y^{l}$, substituting into the participation constraint we have that $y^{l}=\pi y^{l}+(1-\pi) y^{l} \geq \pi p_{2}\left(y^{h}, y^{h}\right)+(1-\pi) p_{2}\left(y^{l}, y^{l}\right)=x$, which is a contradiction with (3).

Next suppose that $d_{2}\left(y^{l}\right)=1$ and $d_{2}\left(y^{h}\right)=0$. Then, the set of payments $p_{2}\left(y^{l}, y^{l}\right)=\phi \leq y^{l}$, $R(\phi)=p_{2}\left(y^{h}, y^{h}\right)=p_{2}\left(y^{l}, y^{h}\right), p_{2}\left(y^{h}, y^{l}\right)=y^{h}, \phi \in\left\lfloor\bar{\phi}, y^{l}\right\rfloor$ satisfies all the constraints above, where $R(\phi)$ is obtained from the participation constraint of the lender

$$
\begin{equation*}
R(\phi)=\frac{x-(1-\pi)(\phi-\mu)}{\pi} \tag{29}
\end{equation*}
$$

and $\bar{\phi}$ is defined such that $x=\pi y^{h}+(1-\pi) \bar{\phi}-\mu$. In other words, $\bar{\phi}$ is the lowest possible payment in the low sate such that the contract is still feasible. Values of $\phi<\bar{\phi} \Rightarrow R(\phi)>y^{h}$, which is not feasible.

Finally, if one sets $d_{2}\left(y^{l}\right)=0$ and $d_{2}\left(y^{h}\right)=1$, then, again, $p_{2}\left(y^{h}, y^{h}\right) \leq y^{l}$ (from the first incentive-compatibility constraint) and the lender will not be able to satisfy its participation constraint.

The variable $\phi$ defines the combination of payments in the low and high states such that the participation constraint of the lender is satisfied. In this sense, the payments are indeterminate. What is crucial, however, is that verification takes place when the low state of the economy is reported. It is in this sense that the contract is a debt contract, independently of the value of $\phi$.

The form of the first-period contract remains to be seen. Given that the second-period contract is the same regardless of the past history, in period one, the expected future utility for the government during the second period is

$$
\begin{equation*}
E V(\phi)=\pi\left[y^{h}-R(\phi)\right]+(1-\pi)\left[y^{l}-\phi\right] \tag{30}
\end{equation*}
$$

and thus, incentive-compatibility constraints are given by

$$
\begin{align*}
& y^{h}-p_{1}\left(y^{h}, y^{h}\right)+E V(\phi) \geq y^{h}-p_{1}\left(y^{h}, y^{l}\right)+E V(\phi)  \tag{31}\\
& y^{l}-p_{1}\left(y^{l}, y^{l}\right)+E V(\phi) \geq y^{l}-p_{1}\left(y^{l}, y^{h}\right)+E V(\phi) \tag{32}
\end{align*}
$$

The participation constraint of the first period lender is as before. Therefore, the problem to solve is the same as in the second period, for there are no reputation effects. Thus, the incentive-compatible contract is again a debt contract with verification in the low state of the economy. QED

Note that $R(\phi)>y^{l}$ for all $\phi \in\left\lfloor\bar{\phi}, y^{l}\right\rfloor$, and that in this contract it is optimal to audit only when $m_{t}=y^{l}$. Why? Because since $R(\phi)=p_{2}\left(y^{h}, y^{h}\right)>y^{l}$ and $y_{t} \in\left\{y^{l}, y^{h}\right\}$, if a government announces $m_{t}=y^{h}$ and repays $R(\phi)$ it must be the case that $y_{t}=y^{h}$ and hence there is no need to audit. If, on the other hand, the contract were to specify $d_{t}\left(m_{t}=y^{l}\right)=0$, then the government would always lie in order to make the smallest payment possible, but this would violate the lender's participation constraint.

In sum, the contract offered for $\phi=G$, at $t=1,2$ is

$$
b_{t}=x ; \quad p_{t}\left(y_{t}, m_{t}\right)=\left\{\begin{array}{ccc}
R(\phi) & \text { if } & m_{t}=y^{h}  \tag{33}\\
\phi & \text { if } & m_{t}=y^{l}
\end{array} ; \quad d_{t}\left(m_{t}\right)=\left\{\begin{array}{lll}
0 & \text { if } & m_{t}=y^{h} \\
1 & \text { if } & m_{t}=y^{l}
\end{array}\right.\right.
$$

where $\phi \in\left\lfloor\bar{\phi}, y^{l}\right\rfloor$, and the contract for $\phi=B, b_{t}=0$.

## V. Unobservable Types

Suppose now that types are not observable. In this case the lender must agree to a contract under uncertainty about the type of government he is dealing with. Note, however, that the bad government will always want to mimic the good one in order to obtain funds to consume. That is, if it lies and offers the same contract that a good government would, then it will receive $b_{t}=x$, of which $\gamma x$ is automatically invested to run the project. Thus, a bad government by claiming to be of the good type ensures itself a consumption of at least $(1-\gamma) x$. On the other hand, by revealing to be a bad type, the government would obtain zero units of the consumption good, for the lender would not finance a bad project (recall assumption (3)).

At the beginning of period $t=1$, the first lender has a prior about the type of government in office being good. Based on this prior, the lender will sign a contract for the first period. The realized variables in this period will be used to update the prior for the next period. The second lender will design a contract based on the updated prior. Let $\rho_{0}$ denote the initial prior that the government is of the good type (thus, the prior about the government being bad is $1-\rho_{0}$ ). After the realization of random variables and conditional on the outcome of events the lender updates his prior with the newly arrived information.

Therefore, update of beliefs, $\rho_{1}$, is the result of a mapping from the public history $h^{1}=\left(g_{1}, d_{1}\right)$ to the unit interval. I model this mapping, or learning process, as Bayesian updating based on the public information available to agents. Thus, learning proceeds as follows:

$$
\begin{equation*}
\rho_{1}=\operatorname{Pr}\left(\theta=G \mid \rho_{0}, g_{1}, d_{1} s_{1}\right)=\frac{\operatorname{Pr}\left(\rho_{0}, g_{1}, d_{1} s_{1} \mid \theta=G\right) \rho_{0}}{\operatorname{Pr}\left(g_{1}, d_{1} s_{1}\right)} \tag{34}
\end{equation*}
$$

For latter reference it is useful to define the following objects. Let $\rho^{G}$ denote the posterior when $m_{1}=y^{l}$ and, upon verification, a good signal is obtained; let $\rho^{B}$ denote the posterior when $m_{1}=y^{l}$ and, upon verification, a bad signal is obtained; and let $\rho^{N}$ be the posterior when $m_{1}=y^{l}$ and no verification takes place. Also, since $y_{t}=y^{h}$ is only possible for the good government, $\rho_{1}=1$ when $m_{t}=y^{h}$. More formally,

$$
\begin{gather*}
\rho^{G}=\frac{(1-\pi) q \rho_{0}}{(1-\pi) q \rho_{0}+(1-q)\left(1-\rho_{0}\right)}  \tag{35}\\
\rho^{B}=\frac{(1-\pi)(1-q) \rho_{0}}{(1-\pi)(1-q) \rho_{0}+q\left(1-\rho_{0}\right)}  \tag{36}\\
\rho^{N}=\frac{(1-\pi) \rho_{0}}{(1-\pi) \rho_{0}+\left(1-\rho_{0}\right)} \tag{37}
\end{gather*}
$$

It can be easily shown that $\rho^{B}<\rho^{N}<\rho^{G}$.

Now we will see that the contract signed during the last period is a debt contract with verification in the low state of the economy. Before proceeding, however, we need to examine the participation constraint of the lender under uncertainty. This is given by

$$
\begin{gather*}
\rho_{1}\left\{\pi\left(p_{2}\left(\rho_{1}, y^{h} ; y^{h}\right)-d_{2}\left(\rho_{1}, y^{h}\right) \mu\right)+(1-\pi)\left(p_{2}\left(\rho_{1}, y^{l} ; y^{l}\right)-d_{2}\left(\rho_{1}, y^{l}\right) \mu\right)\right\}+ \\
\left(1-\rho_{1}\right)\left\{p_{2}\left(\rho_{1}, y^{l} ; y^{l}\right)-d_{2}\left(\rho_{1}, y^{l}\right) \mu\right\} \geq x \tag{38}
\end{gather*}
$$

The first expression in brackets is the expected return for the lender if the government is of the good type, whereas the second expression in brackets is the expected return if the government is of the bad type.

Let $\tilde{\rho}$ denote the minimum prior (i.e. probability assigned to the government being good) such that it induces participation of the lender when verification occurs only in the low state of the economy and with payments $p_{t}\left(\rho_{t-1}, y^{h}, y^{h}\right)=y^{h}$ and $p_{t}\left(\rho_{t-1}, y^{l}, y^{l}\right)=y^{l}$. That is,

$$
\begin{equation*}
\tilde{\rho}=\frac{x-y^{l}-\mu}{\pi\left(y^{h}-y^{l}+\mu\right)} \tag{39}
\end{equation*}
$$

Since $p_{t}\left(\rho_{t-1}, y^{h}, y^{h}\right)=y^{h}$ and $p_{t}\left(\rho_{t-1}, y^{l}, y^{l}\right)=y^{l}$ are the maximum repayments possible, then whenever the prior is lower than the cut-off value $\tilde{\rho}$, the lender will not be able to recoup his investment and thus will set $b_{t}=0$. Let $\bar{\rho}$ denote the value of the prior $\rho_{0}$ for which a low announcement $m_{1}=y^{l}$ and a bad signal $s=B$ result in the posterior being equal to the cut-off point $\tilde{\rho}$.

The following propositions derive the optimal contracts between the government and lenders. Proposition 2 derives the contract for the second period, whereas propositions 3 and 4 establish the contract for period one under different parameter assumptions.

Proposition 2: Given priors $\rho_{1} \in[\tilde{\rho}, 1]$, the feasible incentive-compatible contract between the government and the lender during the second period is a standard debt contract with verification in the low state.

Proof: First recall that the lender can reject a contract if he believes that the government is of the bad type. Thus, in order not to be identified, the bad government will offer the same contract as a good government would. The problem at hand, then, is the same as in the observed-types economy except that now the lender does not know for sure the type of government and hence his participation constraint is given by equation (38).

Incentive-compatibility constraints for the government are

$$
\begin{align*}
& y^{h}-p_{2}\left(\rho_{1}, y^{h}, y^{h}\right) \geq y^{h}-p_{2}\left(\rho_{1}, y^{h}, y^{l}\right)  \tag{40}\\
& y^{l}-p_{2}\left(\rho_{1}, y^{l}, y^{l}\right) \geq y^{l}-p_{2}\left(\rho_{1}, y^{l}, y^{h}\right) \tag{41}
\end{align*}
$$

Making substitutions, it is straightforward to see that maximizing the government's consumption is equivalent to minimizing the function

$$
\begin{equation*}
\rho_{1}\left\{\pi d_{2}\left(\rho_{1}, y^{h}\right) \mu+(1-\pi) d_{2}\left(\rho_{1}, y^{l}\right) \mu\right\}+\left(1-\rho_{1}\right) d_{2}\left(\rho_{1}, y^{l}\right) \mu-\left(1-\rho_{1}\right) p_{2}\left(\rho_{1}, y^{l}, y^{l}\right) \tag{42}
\end{equation*}
$$

subject to the same set of constraints. That is, equations (11) and (12), incentive compatibility (40), (41), and the participation constraint of the lender (38).

As in the proof of proposition 1 , by setting $d_{2}\left(\rho_{1}, y^{h}\right)=d_{2}\left(\rho_{1}, y^{l}\right)=0$ and making substitutions into the participation constraint one obtains that

$$
\begin{equation*}
y^{l} \geq \rho_{1}\left\{\pi p_{2}\left(\rho_{1}, y^{h}, y^{h}\right)+(1-\pi) p_{2}\left(\rho_{1}, y^{l}, y^{l}\right)\right\}+\left(1-\rho_{1}\right) p_{2}\left(\rho_{1}, y^{l}, y^{l}\right)=x \tag{43}
\end{equation*}
$$

which establishes a contradiction (recall that $x>y^{l}$ ). Suppose now that $d_{2}\left(\rho_{1}, y^{h}\right)=1$ and $d_{2}\left(\rho_{1}, y^{l}\right)=0$. Then $p_{2}\left(\rho_{1}, y^{h}, y^{l}\right)=p_{2}\left(\rho_{1}, y^{l}, y^{l}\right)$, and from the IC constraints one has that $p_{2}\left(\rho_{1}, y^{l}, y^{l}\right) \leq y^{l}$. Since $p_{2}\left(\rho_{1}, y^{l}, y^{l}\right)$ and $p_{2}\left(\rho_{1}, y^{h}, y^{h}\right)$ are less or equal than $y^{l}$, the participation constraints of the lender cannot be satisfied. This rules out $d_{2}\left(\rho_{1}, y^{h}\right)=1$ and $d_{2}\left(\rho_{1}, y^{l}\right)=0$ as a possible solution. Finally set $d_{2}\left(\rho_{1}, y^{l}\right)=1$ and $d_{2}\left(\rho_{1}, y^{h}\right)=0$. The participation constraint reduces to

$$
\begin{equation*}
\rho_{1}\left\{\pi p_{2}\left(\rho_{1}, y^{h}, y^{h}\right)+(1-\pi) p_{2}\left(\rho_{1}, y^{l}, y^{l}\right)-\mu\right\}+\left(1-\rho_{1}\right)\left(p_{2}\left(\rho_{1}, y^{l}, y^{l}\right)-\mu\right)=x \tag{44}
\end{equation*}
$$

Note that any pair of payments $p_{2}\left(\rho_{1}, y^{l}, y^{l}\right)=\phi \in\left[0, y^{l}\right]$, $p_{2}\left(\rho_{1}, y^{h}, y^{h}\right)=R_{2}\left(\phi, \rho_{1}\right)=\left[x-\left(1-\rho_{1} \pi\right)(\phi-\mu)\right] / \rho_{1} \pi$ satisfies all the constraints. However, notice that $p_{2}\left(\rho_{1}, y^{l}, y^{l}\right)=y^{l}$ is the value for which the objective function is minimized. In sum, the contract offered is

$$
\begin{gather*}
b_{2}\left(\rho_{1}\right)=x \\
p_{2}\left(\rho_{1}, y_{2}, m_{2}\right)=\left\{\begin{array}{ccc}
R_{2}\left(\rho_{1}\right) & \text { if } & m_{2}=y^{h} \\
y^{l} & \text { if } & m_{2}=y^{l}
\end{array}\right. \tag{45}
\end{gather*}
$$

$$
d_{2}\left(\rho_{1}, m_{2}\right)=\left\{\begin{array}{lll}
0 & \text { if } & m_{2}=y^{h} \\
1 & \text { if } & m_{2}=y^{l}
\end{array}\right.
$$

where $R_{2}\left(\rho_{1}\right)$ is given by

$$
\begin{equation*}
R_{2}\left(\rho_{1}\right)=\frac{x-\left(1-\rho_{1} \pi\right)\left(y^{l}-\mu\right)}{\rho_{1} \pi} \tag{46}
\end{equation*}
$$

For $\rho_{1} \in(0, \tilde{\rho}), b_{2}\left(\rho_{1}\right)=0$. QED
Given that we know the contract for the second period, the expected utility of the government for that period can be expressed as a function of the beliefs $\rho_{1}$

$$
E V\left(\rho_{1}\right)=\left\{\begin{array}{cll}
\pi\left[y^{h}-R_{2}\left(\rho_{1}\right)\right] & \text { if } & \rho_{1} \geq \tilde{\rho}  \tag{47}\\
0 & \text { if } & \rho_{1}<\tilde{\rho}
\end{array}\right.
$$

Now it remains to find the contract for the first period. When types are not observable, the government's messages in the first period will affect the prior, $\rho_{1}$, which in turn will affect the terms of the second-period contract, as seen in (47). For this reason, the government will take into account its continuation payoff, $E V\left(\rho_{1}\right)$, when deciding on the messages to send in $t=1$. This informational link across periods introduces some truth-telling discipline into the government's actions. In other words, the government may refrain from lying in period one if the losses in terms of lower continuation payoff are higher than the gain in the first period. If that is the case, then the lender does not need to verify the output, for the government has enough (reputation) incentives to signal it is of a good type. Proposition 3 shows how there is a range of parameter values for which the reputation effect is not strong enough, and thus, a contract with verification in the low state is optimal. On the other hand, for the complementary range of parameters, it is not optimal to audit when the low state is reported. This is so because, in this case, reporting a low state sets posteriors below $\tilde{\rho}$ and thus $b_{2}=0$. Therefore, it is not profitable for a good government to lie and report $m_{t}=y^{l}$ if $y_{t}=y^{h}$. Only a bad government will report $m_{t}=y^{l}$.

For parameter values satisfying

$$
\begin{gather*}
x-y^{l}>\rho_{0} \pi\left[\pi y^{h}+(1-\pi) y^{l}-x\right]  \tag{48}\\
1-\rho_{0}<\frac{1-\pi}{\pi} \tag{49}
\end{gather*}
$$

we can establish the following result.
Proposition 3: Given priors $\rho_{0} \in[\tilde{\rho}, 1]$, and under conditions (48) and (49), the contract that the government offers to the lender at $t=1$ is a standard debt contract with verification in the low state.

## Proof: See Appendix.

Proposition 4: Given priors $\rho_{0} \in[\tilde{\rho}, 1]$, and if conditions (48) and (49) are not satisfied, the contract that the government offers to the lender at $t=1$ specifies no verification for all states of the economy.

Proof: See Appendix.
For the rest of the paper assume that conditions (48) and (49) hold. Hence, the sequence of contracts offered by the government at $t=1,2$ have the following form

$$
\begin{align*}
& b\left(\rho_{t-1}\right)=x \\
& p_{t}\left(\rho_{t-11}, y_{t}, m_{t}\right)=\left\{\begin{array}{ccc}
R_{t}\left(\rho_{t-1}\right) & \text { if } & m_{2}=y^{h} \\
y^{l} & \text { if } & m_{2}=y^{l}
\end{array}\right.  \tag{50}\\
& d_{2}\left(\rho_{t-1}, m_{t}\right)=\left\{\begin{array}{lll}
0 & \text { if } & m_{2}=y^{h} \\
1 & \text { if } & m_{2}=y^{l}
\end{array}\right.
\end{align*}
$$

where $R_{t}\left(\rho_{t-1}\right)$ is determined from the participation constraint of the lender alive at $t$, and since $d_{1}\left(y^{l}\right)=1, \rho_{0} \in\left\{\rho^{B}, \rho^{G}, 1\right\}$.

For later use, it is useful to see that the participation constraint of the lender

$$
\begin{equation*}
\rho_{t-1}\left\{\pi R_{t}\left(\rho_{t-1}\right)+(1-\pi)\left(y^{l}-\mu\right)\right\}+\left(1-\rho_{t-1}\right)\left(y^{l}-\mu\right)=x \tag{51}
\end{equation*}
$$

delivers

$$
\begin{equation*}
R_{t}\left(\rho_{t-1}\right)=\frac{x-\left(1-\rho_{t-1} \pi\right)\left(y^{l}-\mu\right)}{\rho_{t-1} \pi} \tag{52}
\end{equation*}
$$

As claimed, this contract can be interpreted as a debt contract in which a given amount is repaid to lenders if the state of the economy is good, and in which assets (i.e. $y^{l}$ ) are seized in the event of bankruptcy or default.

Note that for each set of parameters of the model $\left\{\pi, y^{h}, y^{l}, x, \mu\right\}$ there exists $\tilde{\rho}$ such that the participation constraint of the lender (49) with $b_{t}\left(\rho_{t-1}\right)=x$ cannot be satisfied for any $\rho_{t-1} \in(0, \tilde{\rho})$. To see this, first note that in any contract $R_{t}\left(\rho_{t-1}\right) \leq y^{h}$ (otherwise, the contract is clearly not feasible). As $\rho_{t-1} \rightarrow 0$, the expected value of the project for the lender diminishes and eventually becomes less than $x$. In order to ensure participation of the lender, it will be necessary to increase $R_{t}\left(\rho_{t-1}\right)$ as priors deteriorate. Nevertheless, the maximum that the government can offer is $R_{t}\left(\rho_{t-1}\right)=y^{h}$, and so $\tilde{\rho}$ is the lower bound on beliefs for which funding is obtained. The above discussion is summarized in the next proposition.

Proposition 5: If $\rho_{0} \in[0, \tilde{\rho}]$, then $b\left(\rho_{0}\right)=0$.
Proof: Straightforward from the participation constraint of the lender (51).

Lemma 1: The interest rate, $r_{t}\left(\rho_{t-1}\right)=\frac{R_{t}\left(\rho_{t-1}\right)-x}{x}$ is bounded above and below, and evolves inversely with the probability $\rho_{t-1}$ of the government being of the good type.

Proof: Simply note that $R_{t}\left(\rho_{t-1}\right) \in\left[R, y^{h}\right]$, where $R$ is defined by (29) when $\phi=y^{l}$ and stands for the payment asked when there is no uncertainty about types, and $R_{t}=y^{h}$ is the maximum feasible payment that can be asked when priors are very low (i.e. $\rho_{t-1}=\tilde{\rho}$ ). From the participation constraint of lender (51) it is straightforward to see that $\frac{\partial R_{t}}{\partial \rho_{t-1}}=-\frac{x-\left(y^{l}-\mu\right)}{\pi\left(\rho_{t-1}\right)^{2}}<0$ (recall that $x>y^{l}$ ). QED.

The next proposition establishes one of the main results of the paper: in case of default there are instances in which the borrower is able to obtain new funding. In other words, the fact of borrowers having defaulted does not necessarily imply that lenders will deny further credit. If a good signal is received after default it will be optimal for lenders not to punish the defaulting government, but re-lend (see Figure 2). New credit is, nevertheless, easier to obtain for a good government than for a bad government.


Figure 2 : Initial Beliefs

Proposition 6: For all $\rho_{0} \in[\tilde{\rho}, \bar{\rho}]$ the good government obtains new credit with higher probability (i.e., q) than the bad one does (i.e., 1-q).

Proof: Suppose that a government obtains $y_{1}=y^{l}$ and thus must declare bankruptcy. After verification, the lender updates his beliefs. For the good government we have that with probability $q$ the signal will be $s=G$, and beliefs will be updated to $\rho_{1}=\rho^{G}>\tilde{\rho}$. In this case the participation constraint of the new lender is satisfied, and the government is able to obtain new loans. Notice that in this case, by punishing the government and not making a loan the lender guarantees himself a net present value of zero. On the other hand, by accepting a debt contract with the government (given t1he new priors $\rho_{1}$ ) the lender obtains a net present
value of zero (epsilon?), and so it is optimal not to punish and lend. As for the bad government, the good signal is only obtained with probability $(1-q)<q$, since $q>1 / 2$.

Note that if a lender gets a bad signal, $s=B$, lending will cease for both types of government since update of beliefs in this event leads to $\rho_{1}=\rho^{B}<\tilde{\rho}$, and the new lender will set $b\left(\rho_{1}\right)=0$. Why? Because in that case, participation of the lender with priors $\rho_{1}<\tilde{\rho}$ would require $R_{t+1}\left(\rho_{1}\right)>y^{h}$, which is clearly not feasible. QED.

Proposition 7: For all $\rho_{0} \in(\bar{\rho}, 1]$ both governments obtain new credit with probability one.
Proof: Simply note that given that $\rho_{0}>\bar{\rho}$, even if the worst outcome of events takes place, that is, $m_{1}=y^{l}$ and $s=B$, posteriors will be such that the participation constraint of the newborn lender is satisfied ( $\rho_{1}>\tilde{\rho}$ ). QED.

## VI. Concluding Remarks

This paper has provided a framework in which debt arrangements arise as the incentivecompatible contractual response to a situation in which there is private information not only about the realized returns of a project but also about the type of the project itself. Also, it has been studied why, even in the event of default, lenders might find it optimal to provide more funds. Despite its simplicity, I deem that the environment presented above captures some of the main forces at play in the international debt markets of the 19th and early 20th centuries. In my opinion, the most interesting aspect of this simple model is the fact that even in the event of bankruptcies, it might still be possible for a government to obtain new credit. That is, in this environment defaults are not unambiguously detrimental for the defaulting nation. By falling under the scrutiny of international lenders, a good government may be able to signal its worthiness as a future user of funds. As shown, it all depends on the kind of information that lenders are able to gather after defaults. If signals were never informative (i.e., $q=1 / 2$ in the model) we would be in an environment in which default unambiguously damages a country's reputation and thus may lead to denial of fresh credit. However, if some additional information were acquired during defaults, then bankruptcies would not be totally damaging for a good government, for in those instances a government may be able to produce a signal that would distinguish it from a bad government.

A main ingredient of the environment presented in this paper is the assumption that contracts are fully enforceable. Clearly, this element was not present in the international arena during the 19th century. In fact, no legal penalty could be imposed on a nation that repudiated its sovereign debt. Thus, what is it that prevented nations from repudiating their debt? One possible answer is that since these nations were in the midst of a developing process, repudiation and its consequent cut-off from international markets would have imposed very high long-run costs compared to the smaller short-run benefits of repudiating the debt. Therefore, the next step in this line of research is to embed a model similar to the one in this
paper into an environment in which repayment of loans is not enforceable and there are reputation effects.

## Proofs of Propositions 3 and 4

## Proof of Proposition 3:

In this proof, I follow the same strategy as in the proof of proposition 2. I first find the contract that minimizes verification costs, and then examine whether this contract is equivalent to maximizing the government utility subject to the same set of constraints. The problem is, thus, to minimize

$$
\rho_{0}\left\{\pi d_{1}\left(y^{h}\right) \mu+(1-\pi) d_{1}\left(y^{l}\right) \mu\right\}+\left(1-\rho_{0}\right) d_{1}\left(y^{l}\right) \mu-E V\left(\rho_{1}\right)
$$

subject to equation (12), incentive-compatibility constraints of the government (notice that $\rho_{0}$ does not appear in $p_{1}\left(\rho_{0}, y_{1}, m_{1}\right)$ to lessen notation)

$$
\begin{align*}
& p_{1}\left(y^{h}, y^{h}\right)-E V(1) \leq p_{1}\left(y^{h}, y^{l}\right)-\left(1-d_{1}\left(y^{h}\right)\right) E V\left(\rho^{N}\right)-d_{1}\left(y^{l}\right) E V(1)  \tag{A.1}\\
& p_{1}\left(y^{l}, y^{l}\right)-\left(1-d_{1}\left(y^{l}\right)\right) E V\left(\rho^{N}\right)-d_{1}\left(y^{l}\right)\left[q E V\left(\rho^{G}\right)+(1-q) E V\left(\rho^{B}\right)\right] \leq \\
& p_{1}\left(y^{l}, y^{h}\right)-\left(1-d_{1}\left(y^{h}\right)\right) E V(1)-d_{1}\left(y^{h}\right)\left[q E V\left(\rho^{G}\right)+(1-q) E V\left(\rho^{B}\right)\right] \tag{A.2}
\end{align*}
$$

and the participation constraint of the lender
$\rho_{0}\left\{\pi\left(p_{1}\left(y^{h}, y^{h}\right)-d_{1}\left(y^{h}\right) \mu\right)+(1-\pi)\left(p_{1}\left(y^{l}, y^{l}\right)-d_{1}\left(y^{l}\right) \mu\right)\right\}+\left(1-\rho_{0}\right)\left(p_{1}\left(y^{l}, y^{l}\right)-d_{1}\left(y^{l}\right) \mu\right)=x$ (A.3)

Recall that $\rho^{B}<\rho^{N}<\rho^{G}$. We need to consider several cases
Case 1: $\tilde{\rho}<\rho^{B}$
Case 2: $\rho^{N}<\bar{\rho}<\rho^{G}$
Case 3: $\rho^{B}<\tilde{\rho}<\rho^{N}$
(notice that, since $\rho_{0}<\rho^{G}$, the case $\rho^{G}<\tilde{\rho}$ is irrelevant because that would imply $b_{1}=0$.
Case 1: $\tilde{\rho}<\rho^{B}$

Rearranging the incentive-compatibility constraints and taking into account (47) yields

$$
\begin{equation*}
p_{1}\left(y^{h}, y^{h}\right) \leq p_{1}\left(y^{h}, y^{l}\right)+\left(1-d_{1}\left(y^{l}\right)\right)+\pi\left[R\left(\rho^{N}\right)-R\right] \tag{A.4}
\end{equation*}
$$

and

$$
\begin{align*}
& p_{1}\left(y^{l}, y^{l}\right)-p_{1}\left(y^{l}, y^{h}\right)+\pi\left[R\left(\rho^{N}\right)-R\right] \leq \\
& -d_{1}\left(y^{l}\right) \pi\left[q R\left(\rho^{G}\right)+(1-q) R\left(\rho^{B}\right)-R\left(\rho^{N}\right)\right]+d_{1}\left(y^{h}\right) \pi\left[q R\left(\rho^{G}\right)+(1-q) R\left(\rho^{B}\right)-R\right] \tag{A.5}
\end{align*}
$$

Note that we would like to set $d_{1}\left(y^{h}\right)=d_{1}\left(y^{l}\right)=0$ in order to not incur any verification costs. However, in that case, incentive-compatibility constraints would boil down to
$p_{1}\left(y^{h}, y^{h}\right) \leq p_{1}\left(y^{h}, y^{l}\right)+\pi\left[R\left(\rho^{N}\right)-R\right]$
$p_{1}\left(y^{l}, y^{l}\right) \leq p_{1}\left(y^{l}, y^{h}\right)+\pi\left[R\left(\rho^{N}\right)-R\right]$
and it should be the case that $p_{1}\left(y^{h}, y^{h}\right)=p_{1}\left(y^{l}, y^{h}\right)$ and $p_{1}\left(y^{l}, y^{l}\right)=p_{1}\left(y^{h}, y^{l}\right)$ by the informational requirement (12). Making substitutions one can see that (A.6) and (A.7) imply that

$$
\begin{equation*}
p_{1}\left(y^{h}, y^{h}\right)=p_{1}\left(y^{l}, y^{l}\right)+\pi\left\lfloor R\left(\rho^{N}\right)-R\right\rfloor \tag{A.8}
\end{equation*}
$$

We will see, however, that conditions (A.3) and (A.8) cannot all hold simultaneously. Suppose (A.8) holds with equality. Substitute into (A.3) to obtain
$p_{1}\left(y^{l}, y^{l}\right)=x-\rho_{0} \pi^{2}\left[R\left(\rho^{N}\right)-R\right\rfloor$
$p_{1}\left(y^{h}, y^{h}\right)=x+\pi\left(1-\rho_{0} \pi\right)\left[R\left(\rho^{N}\right)-R\right]$
where $R\left(\rho^{N}\right)$ is given in equation (46). After some algebraic manipulations it is easy to see that, under assumption (49), $p_{1}\left(y^{l}, y^{l}\right)>y^{l}$, which is not feasible. In order to lower $p_{1}\left(y^{l}, y^{l}\right)$ to $y^{l}$, one would need to raise $p_{1}\left(y^{h}, y^{h}\right)$ (because of the participation constraint of the lender), but this would violate condition (A.6) and (A.7).

By setting $d_{1}\left(y^{l}\right)=1$ and $d_{1}\left(y^{h}\right)=0$ it is possible to obtain a contract that minimizes verification costs and satisfies incentive-compatibility and participation constraints. This contract is: $d_{1}\left(y^{l}\right)=1, d_{1}\left(y^{h}\right)=0, p_{1}\left(y^{l}, y^{l}\right)=\phi, p_{1}\left(y^{h}, y^{h}\right)=p_{1}\left(y^{l}, y^{h}\right)=R\left(\rho_{0}, \phi\right)$, and $p_{1}\left(y^{h}, y^{l}\right)=y^{h}, \phi \in\left[0, y^{l}\right]$. However, by raising $\phi$ up to $y^{l}$ incentive compatibility constraints are not affected, while the value of the government's objective function can be increased.

Finally, setting $d_{2}\left(\rho_{1}, y^{l}\right)=0$ and $d_{2}\left(\rho_{1}, y^{h}\right)=1$ would, again, imply $p_{1}\left(y^{l} y^{l}\right)=x-\rho_{0} \pi^{2}\left[R\left(\rho^{N}\right)-R-\mu\right]>y^{l}$. Moreover, notice that with a contract that establishes $p_{1}\left(y^{h}, y^{l}\right)=R_{1}\left(\rho_{0}\right)>y^{l}$ verification when $m_{1}=y^{h}$ can be avoided.

Case 2: $\rho^{N}<\bar{\rho}<\rho^{G}$
In this case manipulation of the (IC) constraints yields

$$
\begin{align*}
& p_{1}\left(y^{h}, y^{h}\right) \leq p_{1}\left(y^{h}, y^{l}\right)+\left(1-d_{1}\left(y^{l}\right)\right) \pi\left[y^{h}-R\right]  \tag{A.9}\\
& p_{1}\left(y^{l}, y^{l}\right) \leq p_{1}\left(y^{l}, y^{h}\right)-\left(1-d_{1}\left(y^{h}\right)\right) \pi\left[y^{h}-R\right]-\left[d_{1}\left(y^{h}\right)-d_{1}\left(y^{l}\right)\right] \pi q\left[y^{h}-R\left(\rho^{G}\right)\right] \tag{A.10}
\end{align*}
$$

Again, setting $d_{1}\left(y^{l}\right)=d_{1}\left(y^{h}\right)=0$ delivers
$p_{1}\left(y^{h}, y^{h}\right) \leq p_{1}\left(y^{h}, y^{l}\right)+\pi\left[y^{h}-R\right]$
$p_{1}\left(y^{l}, y^{l}\right) \leq p_{1}\left(y^{l}, y^{h}\right)-\pi\left[y^{h}-R\right]$
and by a similar argument as above one obtains the payments
$p_{1}\left(y^{l}, y^{l}\right)=x-\rho_{0} \pi^{2}\left[y^{h}-R\right]$
$p_{1}\left(y^{h}, y^{h}\right)=x+\pi\left(1-\rho_{0} \pi\right)\left[y^{h}-R\right]$
But $p_{1}\left(y^{l}, y^{l}\right)>y^{l}$ by assumption (48). Thus, following the same reasoning as above, the contract obtained is: $d_{1}\left(y^{l}\right)=1, d_{1}\left(y^{h}\right)=0, p_{1}\left(y^{l}, y^{l}\right)=y^{l}$,
$p_{1}\left(y^{h}, y^{h}\right)=p_{1}\left(y^{l}, y^{h}\right)=R\left(\rho_{0}\right)$, and $p_{1}\left(y^{h}, y^{l}\right)=y^{h}$.
Case 3: $\rho^{B}<\tilde{\rho}<\rho^{N}$
Incentive-compatibility conditions become

$$
\begin{align*}
& p_{1}\left(y^{h}, y^{h}\right) \leq p_{1}\left(y^{h}, y^{l}\right)+\left(1-d_{1}\left(y^{l}\right)\right) \pi\left[y^{h}-R\right]-\left(1-d_{1}\left(y^{l}\right)\right) \pi\left[y^{h}-R\left(\rho^{N}\right)\right]  \tag{A.13}\\
& p_{1}\left(y^{l}, y^{l}\right) \leq p_{1}\left(y^{l}, y^{h}\right)+\left(1-d_{1}\left(y^{l}\right)\right) \pi\left[y^{h}-R\left(\rho^{N}\right)\right]+d_{1}\left(y^{l}\right) \pi q\left[y^{h}-R\left(\rho^{G}\right)\right]- \\
& \left(1-d_{1}\left(y^{h}\right)\right) \pi\left[y^{h}-R\right]-d_{1}\left(y^{h}\right) \pi q\left[y^{h}-R\left(\rho^{G}\right)\right] \tag{A.14}
\end{align*}
$$

Setting $d_{1}\left(y^{l}\right)=d_{1}\left(y^{h}\right)=0$ reduces the (IC) constraints to equations (A.6) and (A.7), with the same results as before. That is, the same debt contract is obtained.

## Proof of Proposition 4:

The proof of proposition 4 is the same as the proof for proposition 3 with only the following difference. In proposition 3 setting $d_{1}\left(y^{l}\right)=d_{1}\left(y^{h}\right)=0$ implied the following payments

Case 1: $\tilde{\rho}<\rho^{B}$

$$
\begin{aligned}
& p_{1}\left(y^{l}, y^{l}\right)=x-\rho_{0} \pi^{2}\left[R\left(\rho^{N}\right)-R\right] \\
& p_{1}\left(y^{h}, y^{h}\right)=x+\pi\left(1-\rho_{0} \pi\right)\left[R\left(\rho^{N}\right)-R\right]
\end{aligned}
$$

The payment $p_{1}\left(y^{l}, y^{l}\right)$ is feasible (i.e. less that $\left.y^{l}\right)$ as long as condition (49) is not satisfied.
Case 2: $\rho^{N}<\bar{\rho}<\rho^{G}$

$$
\begin{aligned}
& p_{1}\left(y^{l}, y^{l}\right)=x-\rho_{0} \pi^{2}\left[y^{h}-R\right] \\
& p_{1}\left(y^{h}, y^{h}\right)=x+\pi\left(1-\rho_{0} \pi\right)\left[y^{h}-R\right]
\end{aligned}
$$

The payment $p_{1}\left(y^{l}, y^{l}\right)$ is feasible as long as condition (48) is not satisfied.
Case 3: $\rho^{B}<\tilde{\rho}<\rho^{N}$

$$
\begin{aligned}
& p_{1}\left(y^{l}, y^{l}\right)=x-\rho_{0} \pi^{2}\left[R\left(\rho^{N}\right)-R\right] \\
& p_{1}\left(y^{h}, y^{h}\right)=x+\pi\left(1-\rho_{0} \pi\right)\left[R\left(\rho^{N}\right)-R\right]
\end{aligned}
$$

The payment $p_{1}\left(y^{l}, y^{l}\right)$ is feasible as long as condition (49) is not satisfied.

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[^1]:    ${ }^{2}$ See Eaton and Gersovitz (1981) for a seminal contribution in this area. More recently, Arellano (2005) and Aguiar and Gopinath (2006) have considered environments in which borrowers suffer exclusion with an exogenous probability of regaining access.

[^2]:    ${ }^{3}$ This loan only brought temporary relief to Bulgaria, as the country faced severe financing needs again in 1901.
    ${ }^{4}$ The rationale behind monetary control was to guarantee the foreign exchange value of the domestic currency, in which repayments of loans were made. For instance, after its 1898 default, Brazil had to accept a deflationary recipe from its bankers. Similarly, in 1902, the National Bank of Bulgaria partially relinquished control over its ability to issue money.

[^3]:    ${ }^{5}$ Certainly, I am abstracting from moral hazard considerations. This does not undermine the point that the paper tries to make: the reaction of lenders to default is not automatically denial of fresh credit, but a verification of the state and, perhaps, renewal of lending.

[^4]:    ${ }^{6}$ In calling this contract a standard debt contract I am following a convention in the costly state verification literature, and more in particular Chang (1990) and Townsend (1979). Throughout the paper, default will be understood as those instances in which the government cannot make a payment high enough to reimburse the lender for the original loan. In other words, default occurs whenever $p_{t}\left(\rho_{t-1}, y_{t}, m_{t}\right)<x$.

